Nonrelativistic Naturalness and the quest for Emergent Lorentz Symmetry

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*Nonrelativistic Short-Distance Completions of a Naturally Light Higgs*, [arXiv:1608.06937] with

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Outline

Relativistic $\phi^4$ theory + Higgs Hierarchy Problem

$z = 2 \phi^4$ theory + nonrelativistic solution to HP

Emergent Lorentz Symmetry

Recap & Outlook
Technical Naturalness and its Successes

Technical naturalness à la ’t Hooft (1979):

“at any energy scale $\mu$, a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system.”

Emergence of QED above $m_e$.

Hadronic resonances in GeV range.

Prediction of the charm quark mass. [Gaillard and Lee, 1974]
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Technical Naturalness and Naturalness Puzzles

Technical naturalness à la Wilson (1971):

“A symmetry breaking term $h_\lambda$ is protected if, in the renormalization group equation for $h_\lambda$, the right-hand side is proportional to $h_\lambda$ or other small coupling constants even when high-order strong, electromagnetic, or weak corrections are taken into account [. . . ].”
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- Cosmological constant problem: $R_{\text{universe}} \gg \ell_{\text{Planck}}$.
- Eta problem: $\eta \ll 1$ or $m_{\text{inflaton}} \ll H$.
- Strange metals: $\rho \sim T$.
- Higgs hierarchy problem: $M_{\text{Higgs}} \ll M_{\text{Planck}}$. 
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Naturalness in $\phi^4$ theory

Relativistic scalar in $3 + 1$ dimensions:

$$S = \frac{1}{2} \int d^4x \left( \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 - \frac{\lambda}{12} \phi^4 \right)$$

EFT below “naturalness scale” $M$

$$\bigcirc \quad \implies \delta m^2 \sim \lambda M^2$$

$$\lambda \sim \varepsilon \iff m^2 \sim \varepsilon M^2$$

$$(\lambda, m^2) \to 0 \implies \phi \to \phi + b \text{ symmetry}$$
The Higgs Hierarchy Problem

- $\lambda \sim 1 \Rightarrow M \sim m \sim 1$ TeV.
- Expect higher dimension ($5, 6, \ldots$) operator effects. [Giudice 1307.7879]
- A sampling of solutions:
  - SUSY (expect superpartners)
  - Conformal (expect second Higgs)
  - Extra dimensions / Braneworld (expect missing energy signals)
- Post-Naturalness Era? [Giudice 1710.07663]
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Nonrelativistic Naturalness in $z = 2 \phi^4$ theory

UV picture: $S = \frac{1}{2} \int dt \, d^3y \left\{ \dot{\phi}^2 - \zeta^2 (\partial_i \partial_j \phi)^2 - c^2 (\partial_i \phi)^2 - m^2 \phi^2 - \frac{\lambda}{12} \phi^4 \right\}$

$z = 2$ scaling symmetry: $y \rightarrow by$ and $t \rightarrow b^2 t$

$\phi \rightarrow \phi + b_{ij} y^i y^j \implies \zeta^2 \sim 1$ and $c^2 \sim \varepsilon_1 M$

$\phi \rightarrow \phi + b \implies \lambda \sim \varepsilon_0 M^{3/2}$ and $m^2 \sim \varepsilon_0 M^2$ with $\varepsilon_0 \ll \varepsilon_1 \ll 1$
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**IR picture:**

$$S = \frac{1}{2} \int d^4x \left\{ (\nabla \mu \Phi)^2 - m^2 \Phi^2 - \frac{\lambda_h}{12} \Phi^4 - \tilde{\zeta}^2 (\nabla_i^2 \Phi)^2 \right\}$$

$x^0 = t$, $x^i = \frac{y^i}{c}$, $\Phi = c^{3/2} \phi$, $\lambda_h = \frac{\lambda}{c^3}$, $\tilde{\zeta}^2 = \frac{\zeta^2}{c^4}$ and $\nabla \mu = \frac{\partial}{\partial x^\mu}$

**Good:** $\lambda_h = \frac{\lambda}{c^3} \sim \frac{\epsilon_0 M^{3/2}}{\epsilon_1^{3/2} M^{3/2}} = \frac{\epsilon_0}{\epsilon_1^{3/2}} \sim 1$

**Bad:** $\tilde{\zeta}^2 = \frac{\zeta^2}{c^4} \sim 1$
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Bad: $\tilde{\zeta}^2 = \frac{\zeta^2}{c^4} \sim \frac{1}{\epsilon_1^2 M^2} = \frac{\epsilon_0}{\epsilon_1^2 m^2} \sim \frac{1}{\epsilon_1^{1/2} m^2}$
Nonrelativistic Solution to HP

\[ S = \frac{1}{2} \int dt \, d^3 y \left\{ \dot{\phi}^2 - \zeta_3^2 (\partial^3 \phi)^2 - \zeta_2^2 (\partial^2 \phi)^2 - c^2 (\partial \phi)^2 - m^2 \phi^2 - \frac{\lambda}{12} \phi^4 \right\} \]

\[ \zeta_3^2 \sim 1, \quad \zeta_2^2 \sim \varepsilon_2 M^{2/3}, \quad c^2 \sim \varepsilon_1 M^{4/3} \]

\[ \lambda \sim \varepsilon_0 M^2, \quad m^2 \sim \varepsilon_0 M^2, \quad \varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1. \]
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\[ \lambda_h \sim \frac{\varepsilon_0}{\varepsilon_1^{3/2}} \sim 1, \quad \tilde{\zeta}_3^2 \sim \frac{1}{m^4}, \quad \tilde{\zeta}_2^2 \sim \frac{\varepsilon_2}{\varepsilon_1^{1/2}} \frac{1}{m^2} \implies \frac{\varepsilon_2}{\varepsilon_1^{1/2}} \leq 1 \]
Yukawa Interactions

Assume relativistic fermions

UV: $Y_f \phi \bar{\psi} \psi \rightarrow$ IR: $y_f \Phi \bar{\psi} \psi$

\[
\delta m^2 \sim Y_f^2 \Rightarrow Y_f \sim \varepsilon_0^{1/2} M
\]

\[
y_f = \frac{Y_f}{c^{3/2}} \sim \frac{\varepsilon_0^{1/2}}{\varepsilon_1^{3/4}} \sim 1 \Rightarrow \text{accommodates fermion masses}
\]

e.g., vacuum Čerenkov radiation of fermion if \( \frac{d\omega}{dk} \leq c \)

Conservative lower bound for Čerenkov onset in our model \( \sim 100 \) TeV

Cosmic ray electrons at \( \sim 5 \) TeV directly observed (VERITAS)

Indirectly at \( \lesssim 100 \) TeV from supernovae x-ray synch rad (ASCA)
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Lorentz Invariance in the IR

Need same $c$ for ALL species in the IR!

Force $c$ for all species to be $c_0$ in UV.

$$\beta_i = \left( -\frac{4}{3} + \# \frac{e^4}{c_0^2} \right) \rho_i$$

where $c_i^2 = \rho_i M^{4/3}$

$$\sim \varepsilon_1 \ll 1$$

Matching of $c$'s is preserved in the IR up to $\varepsilon_1 \log(M/m) \ll 1$

EFT cannot explain the “initial” condition of $c_0$
Emergent Lorentz Symmetry

Scalar and spinor (speeds $c_b$ and $c_f$) + Yukawa $g$. [Anber and Donoghue 1102.0789]

$\beta_{c_b}$ or $c_f \propto (c_b - c_f) g^2$ and $\beta_g \sim g^3$.

Generic $g$: Lorentz symmetry emerges (slowly) at low energies.

Strongly coupled fixed point $g_*$: $c_b - c_f \to 0$ as a power law!

Favorable evidence in holography. [Bednik, Pujolàs and Sibiryakov 1305.0011]

Does such a strongly coupled fixed point exist in our theory?
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Shift symmetries produce natural hierarchies among couplings.

Mass hierarchy even with $O(1)$ IR couplings.

Accommodate SM Yukawas and Yang-Mills couplings naturally.

More experimental bounds.

Explore decay of LIV after strong coupling.

Apply to other problems (e.g., strange metals).
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Thank you!